# ADMIXTURE TRANSPORT INSIDE A CYLINDER AT A VARIABLE STREAM VELOCITY AND DURING A TRANSIENT GAS TRANSFER TO THE WALL

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Results are shown of a theoretical and experimental study concerning the convective–diffusive transport of a sorbed admixture in a channel where the stream velocity varies with time and where a transient transfer of gas to the wall occurs.

Earlier in [1] the authors have analyzed the transport of a sorbed admixture inside a cylinder at a constant stream velocity and during a free transfer of gas to the wall. This condition prevails in practice during a short-time (2-5 h) exhaust of admixture.

During the extinguishing of underground fires by the isolation method, sorbed admixtures are fed to appropriate areas often for 10-20 h or even longer periods. In this case the stream velocity in the borehole (channel) and the rate of admixture absorption usually decrease while the concentration of admixture in the stream increases. Evidently, exhaust gas begins to saturate the borehole walls which confine the stream. The problem can be formulated as follows [2]:

$$\psi\left[\frac{\partial^2 c}{\partial x^2}+\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)\right]=u(t)\frac{\partial c}{\partial x}+\frac{\partial c}{\partial t}; \tag{1}$$

$$c(x,\ r,\ 0)=c_1;\qquad c(0,\ r,\ t)=f(t);$$
$$c(\infty,\ r,\ t)=c_1;\quad \left.\frac{\partial c}{\partial r}\right|_{r=a}+\alpha c|_{r=a}=\alpha F(x,\ t);\quad F(\infty,\ t)=c_1. \tag{2}$$

Function F(x, t) describes the kinetics of sorbed gas accumulation in the borehole walls. According to experimental evidence, for practical purposes

$$F(x,\ t)=g(t)\,e^{-\nu x} \tag{3}$$

within sufficient accuracy.

Function g(t) describes the time rate of admixture diffusion or filtration as well as the sorption rate. Functions f(t) and g(t) are both bounded.

We will assume that, according to [3], the coefficient of turbulent mixing is a function of the stream velocity and that, for simplicity, it is a linear function

$$\psi=\beta u(t). \tag{4}$$

Such an assumption is entirely acceptable for engineering purposes.

Then, after substituting in (1) and (2) $x = a\xi$, $r = a\rho$, $\tau = (\beta/a^2)\int_0^t u(t)dt = z(t)$, $t = z^{-1}(\tau)$, then letting $c^*(x, r, t) = c(x, r, t) - c_1$ and performing a Laplace transformation with respect to variable $\tau$, we finally have

$$\frac{\partial^2 G}{\partial \xi^2}+\frac{1}{\rho}\cdot\frac{\partial}{\partial \rho}\left(\rho\frac{\partial G}{\partial \rho}\right)=v\frac{\partial G}{\partial \xi}+pG, \tag{5}$$

$$G(0,\ \rho,\ \tau)=\Phi(p);\quad G(\infty,\ \rho,\ \tau)=0;\qquad \left.\frac{\partial G}{\partial \rho}\right|_{\rho=1}+AG|_{\rho=1}=\overline{R}(p)\,e^{-\nu a\xi}, \tag{6}$$

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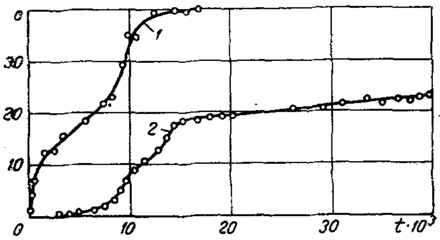


Fig. 1. Theoretical curves and test values of gaseous carbon dioxide concentration (%) as a function of time (sec), at the entrance (1) and at 350 m from the entrance (2).

where $A = a\alpha$ and G, $\Phi(p)$, $\bar{R}(p)$ are the transforms of functions c, f(t), $R(\tau)$ respectively, with $R(\tau) = R[z(t)] = a\alpha g(t)$.

Applying to Eq. (5) the Hankel integral transformation $\int_0^1 G\rho J_0(\lambda\rho)d\rho = q$ and letting $-\lambda J_0'(\lambda)/J_0(\lambda) = A$ yields

$$\frac{d^2q}{d\xi^2} - (\lambda^2 + p)q - v\frac{dq}{d\xi} = -\bar{R}(p)e^{-va\xi}J_0(\lambda), \tag{7}$$

whose solution is

$$q = \bar{c}_2 \exp\left[\left(\frac{v}{2} - \sqrt{\frac{v^2}{4} + \lambda^2 + p}\right)\xi\right] - \frac{\bar{R}(p)J_0(\lambda)e^{-va\xi}}{v^2a^2 - (\lambda^2 + p) + vav}. \tag{8}$$

On the basis of the Hankel transformation, function G can be represented in terms of a series

$$G = \sum_{n=1}^{\infty}\left\{\bar{c}_n \exp\left[\left(\frac{v}{2} - \sqrt{\frac{v^2}{4} + \lambda_n^2 + p}\right)\xi\right] + \frac{\bar{R}(p)J_0(\lambda)e^{-va\xi}}{v^2a^2 - (\lambda^2 + p) + vav}\right\}, \tag{9}$$

where $\lambda_n$ are the positive roots of the equation $\lambda J_1(\lambda) - AJ_0(\lambda) = 0$.

Having determined coefficients $\bar{c}_n$ and inverse transforming to the original function, we obtain

$$c_n = \frac{2F(\tau)\int_0^1 \rho J_0(\lambda_n\rho)\,d\rho}{J_1^2(\lambda_n) + J_0^2(\lambda_n)} + J_0(\lambda_n)\int_0^{\tau} R(\tau - \theta)\exp\left[-(\lambda_n^2 + v^2a^2 + vav)\,\theta\right]d\theta. \tag{10}$$

Applying once more the convolution principle and changing to the original variables, we have

$$c = \sum_{n=1}^{\infty}\left\{\int_0^z\left[\left(\frac{2[f(\mu) - c_1]J_1(\lambda_n)}{\lambda_n J_1^2(\lambda_n) + \lambda_n J_0^2(\lambda_n)} - J_0(\lambda_n)\int_{z(t)-2z(\mu)}^{z(t)-z(\mu)} a\alpha g(s)\exp\left[\left(-\lambda_n^2 + v^2a^2 + \frac{a^2v}{\beta}\right)\left(z(t) - z(\mu) - z(s)\right)\right)z'(s)ds\right.\right.$$
$$\times\frac{\xi}{2\left[z(t) - z(\mu)\sqrt{\pi(t) - z(\mu)}\right]}\exp\left[-\frac{a^2}{4\beta^2} + \lambda_n^2\left(z(t) - z(\mu)\right) - \frac{\xi}{4\left(z(t) - z(\mu)\right)} + \frac{a}{2\beta}\xi\right]\Bigg]z'(\mu)\,d\mu$$
$$\left. + J_0(\lambda_n)e^{-va\xi}\int_0^{z(t)} a\alpha g(\mu)\exp\left[\left(-\lambda_n^2 - v^2a^2 + \frac{a^2v}{\beta}\right)(z(t) - z(\mu))\right]z'(\mu)\,d\mu\right\}J_0(\lambda_n\rho) + c_1, \tag{11}$$

where

$$z'(t) = \frac{\beta}{a^2}u(t), \quad z(t) = \frac{\beta}{a^2}\int_0^t u(t)\,dt.$$

Series (11) is a fast converging one. The first five terms were sufficient in our calculations, inasmuch as an accuracy adequate for engineering purposes ($\sigma < 1\%$) was thus achieved. The integrals were evaluated by the Gauss method; the maximum number of nodes was 40. The original function f(t) was constructed from test points by linear interpolation.

The results of theoretical calculations performed according to formula (11) on a "URAL-4" computer were checked against special measurements under field conditions.

During the experiment, gaseous carbon dioxide was exhausted at a 0.2 $m^3$/sec rate through the entrance (x = 0) into the ventilating duct along the borehole. At various distances from the entrance, air was sampled at several points and then analyzed in the laboratory.

The carbon dioxide concentration at the entrance varied along curve 1 in Fig. 1. The mean-over-the-section stream velocity varied according to the relation

$$u = 0.03 + 0.06e^{-0.05 \cdot 10^3 t} \tag{12}$$

The theoretical curve 2 of carbon dioxide concentration at 350 m from the exhaust point, calculated according to formula (11), is shown in Fig. 1 along with the test values. The maximum difference between both is 10% – entirely acceptable for practical purposes.

## NOTATION

| | |
|---|---|
| c | is the admixture concentration, %; |
| r, x | are the space coordinates, m; |
| t | is the time, sec; |
| $a$ | is the radius of cylinder, m; |
| $\psi$ | is the coefficient of turbulent mixing, $m^2/sec$; |
| u | is the stream velocity, m/sec; |
| f(t) | is the function of concentration at the entrance (x = 0), %; |
| $\alpha = A/\varphi$; | |
| A | is the admixture sorption per unit surface area, $m^3/sec \cdot m^{-2}$; |
| $\varphi$ | is the diffusivity from stream to wall, $m^2/sec$. |

## LITERATURE CITED


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